Development of Best Beta CAPM with Adjustment of Sharia Elements: A Case Study on Sharia Stocks in Indonesia

A Aziz[1,*], Supriyanto[2] and Abdurakhman[3]

[1] Department of Mathematics, Jendral Soedirman University, Purwokerto 53122, Indonesia
[3] Department Mathematics, Universitas Gadjah Mada, Yogyakarta 55281, Indonesia

*Corresponding author's email: abdul.math@unsoed.ac.id

Abstract. This paper introduces the Best Sharia-based Capital Asset Pricing Model (BSCAPM), a modification of the BCAPM model integrating Islamic finance principles. This study focuses on optimizing the beta parameter within the model by integrating Sharia-compliant factors such as zakat and purification, while excluding short-selling practices. Using data from the Jakarta Islamic Index (JII) from June 2020 to November 2024, the BSCAPM portfolio outperforms the BCAPM portfolio in terms of the Sharpe ratio. The findings indicate that the BSCAPM serves as a viable alternative framework for Islamic investment modelling, providing Muslim investors with a Sharia-compliant, optimal portfolio formation model. The research contributes to the underexplored domain of portfolio selection modelling in the Islamic sector, enriching references on asset pricing in Sharia portfolios, particularly in the Indonesian Sharia stock market.
Keyword: Best Beta, Islamic stocks, Portfolio, Purification, SCAPM, Zakat.

1. Introduction

In the era of globalization, capital markets have developed rapidly worldwide, including in Indonesia. Based on data obtained from the IDX, the number of publicly listed stock securities in Indonesia as of December 2, 2024, was 941, spanning various sectors. Among the numerous stocks available, investors often consider two primary factors: potential profits (returns) and the risks associated with uncertainty.

In the CAPM framework, beta is employed as a measure of systematic risk. Beta is widely utilized for two reasons. First, it enhances total risk measures by avoiding reliance on variance and standard deviation. Second, various empirical studies suggest that beta is relatively stable. In this study, we introduce a slight variation to the beta in the Capital Asset Pricing Model (CAPM) by incorporating risk as the second moment of the portfolio return around the target return. The analysis aims to improve model accuracy by minimizing CAPM pricing errors. By fitting US historical data into the model, it was found that optimizing beta could improve the pricing accuracy of the CAPM by 20 % to 30 % per year [1].

Although CAPM can enhance pricing accuracy, it has inherent shortcomings in practice due to its assumptions and limitations as a single-factor model. Consequently, numerous authors have endeavored to develop and adapt CAPM to real-world conditions. The presence of interest rates is evident in lending and borrowing activities, along with their associated risks. Indeed, during periods of instability, risks may either rise or fall. This phenomenon highlights the emergence of Sharia-based finance as an alternative to conventional systems.

In alignment with the growing trend of Islamic investment, many individuals are transitioning to the Islamic system. Several empirical studies suggest that Islamic finance exhibits higher stability and reliability compared to its conventional counterpart. This perspective may be influenced by evidence indicating that Islamic banking yields positive results in three critical aspects: profitability, efficiency, and liquidity. In fact, research by Parashar reported that even in crisis conditions, Islamic banking demonstrates greater resilience compared to conventional banking [2]. Additionally, Islamic Banking (IB) has proven to be more profitable than conventional banking (CB) based on research results spanning from 2007 to 2010.

Some researchers agree that the Capital Asset Pricing Model (CAPM) can be applied in Islamic finance [3], [4], [5]. One of the elements in CAPM requires a risk-free interest rate, commonly known as the risk-free return. Nevertheless, the concept of a risk-free interest rate is incongruent with Sharia, as Islam prohibits the payment or receipt of interest. El-Ashker extends the traditional Capital Asset Pricing Model (CAPM) theoretical framework by replacing the risk-free rate of return with a zakat rate of 2.5 % in asset pricing [6]. This adjustment is cited as a valuable reason for Islamic investors to consider investment. Recent Islamic CAPM modelling by Derbali et al. and Hazny et al. [7] attempted to include *sukuk* (Islamic bonds compliant with Sharia principles), *zakat* (mandatory almsgiving in Islam), and purification in the process. Sukuk from the Islamic Development Bank (IDB) can be a novel breakthrough in Islamic investment, despite the absence of a risk-free interest rate. An important distinction arises between sukuk yields and conventional interest rates. Sukuk structures are designed to circumvent the religious prohibition of usury. In a traditional bond structure, the issuer pays interest to the bondholder [8]. Sukuk can be an investment in various types of assets, particularly in ventures devoid of prohibited *riba* components and non-halal businesses. This modified version of the CAPM is herein termed the Shariah-Compliant Asset Pricing Model (SCAPM). The three principles included in this model are the replacement of interest rates with sukuk, the incorporation of zakat deduction, and the consideration of purification.

In this study, CAPM will be modified using two methods simultaneously to obtain an enhanced portfolio formation model. The first modification aims to optimize beta, thereby minimizing pricing errors. The second involves replacing interest rates and includes Sharia elements related to zakat, sukuk, and purification.

2. Literature Review

2.1. Best Beta Capital Asset Pricing Model

Assuming multivariate normality, the risk-efficient optimal portfolio achieves the highest Sharpe ratio among all possible asset combinations. Additionally, the multivariate normal distribution indicates that the two-fund split holds for all investors only if each asset's return satisfies the two-fund separation condition.

In order to optimize the Best Beta Capital Asset Pricing Model by minimizing pricing errors, we can employ the following density function:

$$f(x_t \mid x_{m,t}) = (2\pi)^{N/2} \mid \Sigma \mid^{-\frac{1}{2}} e^{\left\{-\frac{1}{2}(x_t - \beta x_{m,t})^T \Sigma^{-1} (x_t - \beta x_{m,t})\right\}} \tag{1}$$

where x_i denotes the observed return of the *i*-th risky asset, β is the vector of systematic risk coefficients, and Σ represents the covariance matrix of asset returns. The above density function expresses the joint distribution of asset returns, which is assumed to follow a multivariate normal distribution in the BCAPM framework. The above equation is estimated using the Maximum Likelihood Estimation (MLE) method with the likelihood function as follow:

$$\begin{aligned} L(\beta, \Sigma) &= \prod_{t=1}^{T} f(x_t \mid x_{m,t}) \\ &= \prod_{t=1}^{T} (2\pi)^{-\frac{N}{2}} \mid \Sigma \mid^{-\frac{1}{2}} e^{\left\{-\frac{1}{2}(x_t - \beta x_{m,t})^T \Sigma^{-1} (x_t - \beta x_{m,t})\right\}} \end{aligned}$$

$$= (2\pi)^{-\frac{NT}{2}} |\Sigma|^{-\frac{T}{2}} e^{\left\{\sum_{t=1}^{T} -\frac{1}{2}(x_t - \beta x_{m,t})^T \Sigma^{-1} (x_t - \beta x_{m,t})\right\}}.$$

By taking the natural logarithm of the likelihood function and differentiating with respect to β and Σ, their estimators are derived:

$$\hat{\beta} = \frac{\sum_{t=1}^{T} x_t x_{m,t}}{\sum_{t=1}^{T} x_{m,t}^2} \tag{2}$$

where L is the log-likelihood function of the asset return distribution, and :

$$\Sigma^{-1} = \frac{1}{\left[\sum_{t=1}^{T} (x_t - \beta x_{m,t})(x_t - \beta x_{m,t})^T\right]}. \tag{3}$$

The role of the best beta, denoted as β^B is crucial in minimizing mispricing. Utilizing the aforementioned estimation process, we obtain the optimal beta for the CAPM model:

$$\beta^B = \frac{E(x_m, x)}{E(x_m^2)} = \frac{cov(x_i, x_m) + E(x_i, x_m)}{var(x_i) + [E(x_i)]^2}. \tag{4}$$

The Best Beta CAPM (BCAPM) seeks to improve the predictive accuracy of asset returns relative to the traditional CAPM.

2.2. CAPM vs BCAPM

The Best Beta Capital Asset Pricing Model (BCAPM) is a simplified version of the CAPM. Its second equation can be employed to predict the linear relationship between risky assets $E(X)$ and β, as expressed in the equation:

$$E(X) = \beta E(X_m), \tag{5}$$

where R_i represents the return of the i-th asset and R_m represents the market return, and beta can be expressed as:

$$\beta = \frac{cov(R_i, R_m)}{var(R_m)} = \beta^{MV}. \tag{6}$$

In the CAPM model, investors share the same perception of risk, as measured by the variance of returns, but they may differ in their views on the degree of risk aversion. Nevertheless, each investor chooses the same optimal risk portfolio. Thus, the model is redefined as the CAPM model with the new beta measure:

$$\beta^B = \frac{E(X_m X)}{E(X_m^2)}, \tag{7}$$

where the letter 'B' originates from the term 'best beta,' representing an enhancement of the beta in the preceding CAPM model. Its role involves minimizing the potential pricing errors, symbolized by the notation α.

To further facilitate a comparative analysis of pricing errors, the expected return prediction models for both the CAPM and BCAPM are defined as follows:

$$E^{MV}(X_i) = \beta^{MV} E(X_m) \tag{8}$$

$$E^{B}(X_i) = \beta^{B} E(X_m). \tag{9}$$

Furthermore, a pricing error measure for both CAPM and BCAPM is defined as follows:

$$\alpha^{MV} = E(X_i) - E^{MV}(X_m) \tag{10}$$

$$\alpha^{B} = E(X_i) - E^{B}(X_m). \tag{11}$$

The single index model can be employed to simplify the calculation process. To provide a more detailed explanation, the known relationship between X_i and X_m can be expressed as:

$$X_i = a_i + b_i X_m + \varepsilon_i, \quad i = 1, 2, \cdots, n \tag{12}$$

with a_i and b_i is fixed real constants, and the random variable ε_i with $E(\varepsilon_i) = 0$, Equation (12) can be rewritten without further restrictions as:

$$\varepsilon_i = X_i - a_i - b_i X_m \quad ; i = 1, 2, \cdots, n \tag{13}$$

where

$$a_i = E(X_i) - b_i(X_m) \quad ; i = 1, 2, \cdots, n \tag{14}$$

for any b_i.

In the CAPM model, minimizing the pricing error involves minimizing $E(\varepsilon_i) = 0$, or it can be expressed as:

$$\min_{a_i, b_i} E(X_i - a_i - b_i X_m)^2, \tag{15}$$

by finding the partial derivatives with respect to a_i and b_i, we obtain the solution:

$$a_i = E(X_i) - \beta_i^{MV} E(X_m) = \alpha_i^{MV}. \tag{16}$$

On the other hand, BCAPM identifies uncertainty in the pricing error and aims to minimize $E(a_i + \varepsilon_i)^2$ as the new measure of pricing error, which can be expressed as:

$$\min_{b_i} E(X_i - b_i X_m)^2 \tag{17}$$

by finding the partial derivative of b_i and setting it equal to 0, we obtain:

$$a_i = E(X_i) - \beta_i^{B} E(X_m) = \alpha_i^{B}. \tag{18}$$

Equations (16) and (18) demonstrate that the constant a_i is the source of pricing errors for the CAPM and BCAPM. By minimizing $E(a_i + \varepsilon_i)^2$ in comparison to $E(\varepsilon_i)^2$, we achieve the smallest pricing error. The following theorem compares the relative pricing accuracy of the two models presented in Equations (10) and (11).

Based on the discussion, it appears that BCAPM can predict pricing errors more accurately than CAPM. The theorem demonstrates that the relationship between expected return and beta, as predicted in the CAPM model, can be enhanced. Furthermore, the theorem emphasizes the significance of beta selection in measuring the systematic risk of an asset.

2.3. Portfolio Weight Formation

Investors combine a risky asset with a risk-free asset. An asset is considered risk-free when the future return is certain. One example of a risk-free asset is government-issued bonds. In the case of Indonesia, Bank Indonesia Certificates (SBIs) issued by Bank Indonesia (the central bank of Indonesia) serve as an example of a risk-free asset.

The portfolio weighting formula can be solved by minimizing the variance $(w^T \Sigma\, w)$ with the constraint $w^T(\mu - R_f\, \mathbf{1}) = \mu_p - R_f$, which can be expressed as:

$$\begin{aligned} &\min \quad w^T \Sigma w \\ &\text{s.t} \quad w^T \mu = \mu_p \\ &\text{and} \quad w^T 1 = 1 \end{aligned} \tag{19}$$

By using the Lagrange function, the weighting equation for BCAPM is obtained.

$$w_i = \frac{\Sigma^{-1}(\mu - R_f \mathbf{1})}{(\mu - R_f)\Sigma^{-1}\mathbf{1}^T} \tag{20}$$

where R_f is risk free rate, Σ is variance covariance matrix, and $\mathbf{1}$ is vector with all its elements are set to one.

2.4. Shariah-Compliant Asset Pricing Model with Zakat and Purification

Several types of research related to Islamic investment principles have emerged [9]. Some countries are particularly focused on Shariah compliant financial products. For instance, sukuk is presented as an alternative asset to replace conventional bonds within the Shariah capital market, differing from traditional bonds [10]. The adjustment in the Capital Asset Pricing Model (CAPM) involves incorporating the return rate of sukuk rather than the interest rate, aligning with Islamic principles that emphasize the association of profitability with risk [4]. The evolution of CAPM in Shariah principles includes factors such as zakat reduction and purification in the computation of investment returns [7], [8]. The proposed model advocates using the yield level of sukuk assets instead of the risk-free level. Consequently, in this Shariah-adjusted model, consideration must be given to zakat obligations and the incorporation of purification from the outset of calculating expected returns [11].

Two forms of investment reduction involve zakat and purification. Zakat, obligatory for Muslims when specific criteria are met, functions as a tax that serves to purify the wealth of Muslims. However, when engaging in investment activities, it is essential to consider the presence of prohibited activities. Therefore, purification becomes a crucial step in ensuring the ethical reception of profits. The designated zakat amount in this study is set at 2.5 %. Purification, on the other hand, involves calculating the percentage of total income derived from non-halal sources. In Indonesia, assets chosen for inclusion in the Indonesian Sharia Stock Index (ISSI) or the Islamic Composite Index must be listed on the Indonesia Stock Exchange (IDX). In this procedure, the concept of purification is distinct from zakat. The proportion of non-halal income to total revenue must not exceed 10 %. The National Sharia Board-Indonesian Council of Ulama (DSN-MUI) established this guideline for classifying Shariah-compliant stocks. As per Hazny et al. [12], this purification process is represented by δ_i.

$$\delta_i = \frac{\text{non-halal income}}{\text{total income}} \tag{21}$$

Subekti et al. revises the adjusted return considering zakat and purification expenses. To differentiate between conventional and Islamic portfolio notations, the anticipated return for the new Islamic portfolio is denoted as $E(R_{ps})$. This study operates under the limiting assumption that, in future periods, the projected portfolio return is redefined, incorporating deductions for zakat and vigilance regarding non-halal income. The anticipated return and variance of each individual stock are Sharia-compliant (i), incorporating adjustments for zakat (z) and purification (δ) factors [13].

Assuming that all investors take into account the zakat (z) and purification (δ_i) factors, expected return of the portfolio is calculated as follows:

$$\mu_{ps} = (1-z)\left(\sum_{i=1}^{n} w_i(1-\delta_i)\mu_i\right). \tag{22}$$

Equation (22) represents the portfolio's novel anticipated return, while the portfolio variance is expressed as:

$$\sigma_{ps}^2 = \sum_{i=1}^{n} w_i^2(1-z)^2(1-\delta_i)^2\sigma_i^2 + \sum_{i=1}^{n}\sum_{j\neq i}^{n} w_i w_j (1-z)^2(1-\delta_i)(1-\delta_j)\sigma_{ij} \tag{23}$$

Furthermore, in consideration of the zakat and purification factors, the Sharpe ratio equation, as defined by the measure of portfolio performance, is expressed as:

$$\theta_{SR} = \frac{\mu_{ps} - (1-z)R_s}{\sigma_{ps}} \tag{24}$$

where θ_{SR} represents the Sharpe ratio of the Shariah-compliant portfolio, R_s denotes the sukuk rate used as the Shariah-compliant equivalent of the risk-free rate. This ratio measures the excess return per unit of risk under Islamic investment principles. Hence, the updated goal is to derive the SCAPM equation by maximizing the portfolio's new Sharpe ratio. This involves finding the derivative of the aforementioned equation above and acquiring the result:

$$\mu_i = \left[\frac{1}{1-\delta_i}\right]R_s + \beta_i\left[\mu_M - \frac{R_s}{1-\delta_M}\right] \tag{25}$$

the purification factor for the market, denoted as δ_M , plays a crucial role in enhancing market purification. Achieving market purification involves introducing a novel definition for the Sharpe Ratio concerning market returns. As a result, the objective function for the market portfolio is expressed through the subsequent equation:

$$\theta^* = \frac{(1-z)(1-\delta_M)\mu_M - (1-z)R_s}{\sigma_M} \tag{26}$$

This results in the formulation of the equation for market purification rate as:

$$\delta_M = 1 - \frac{\theta^*\sigma_M + (1-z)R_s}{(1-z)\mu_M} \tag{27}$$

3. Data and Methodology

3.1. Data

This research utilizes data pertaining to the shares of publicly listed companies within the Jakarta Islamic Index (JII) 70 stock group. The selection of JII 70 stocks is based on available stock data from the desired year, duly published. The study sources its information on the JII 70 stocks from the Indonesia Stock Exchange (IDX) Statistical Report. The investigation tracks the progression of JII 70 stocks over a span of 3 years, divided into seven periods. The data utilized consists of the weekly closing prices of 70 stocks from the JII index, spanning from June 1, 2020, to November 30, 2024. Stocks within the JII 70 were selected using purposive sampling based on specific eligibility criteria. Firstly, stocks consistently present in the JII 70 throughout the observation period were included, resulting in 41 selected stocks. Secondly, among the consistently included stocks, those exhibiting a positive mean return value were chosen, leading to the selection of 28 stocks. Additionally, the chosen stocks were required to demonstrate normal distribution and stationarity. Stationarity testing was essential to ensure that regression analysis was performed on stationary time series data. The outcomes of the normality and stationarity tests for stocks exhibiting positive returns are depicted in Table 1

Table 1. *Jarque Bera* (JB) and *Augmented Dickey Fuller* (ADF) test.

Stock	Company	Mean	JB test	ADF Test
ADRO	Astra International	0.00655	0.0015	0.01
AKRA	AKR Corporindo	0.00697	0.0000	0.01
ANTM	Aneka Tambang	0.00652	0.0000	0.01
ASII	Astra International	0.00198	0.3943	0.01
BMTR	Global Mediacom	0.00176	0.0000	0.01
BRIS	Bank Syariah	0.00909	0.0000	0.01
CTRA	Ciputra Development	0.00391	0.0000	0.01
DMAS	Puradelta Lestari	0.00274	0.0000	0.01
ERAA	Erajaya Swasembada	0.00258	0.0000	0.01
ICBP	CBP Sukses	0.00167	0.002	0.01
INCO	Vale Indonesia	0.00271	0.0000	0.01
INDF	Indofood Sukses	0.00125	0.0209	0.01
ISAT	Indosat	0.00935	0.0000	0.01
ITMG	Indo Tambangraya	0.00947	0.0000	0.01
JPFA	Japfa Comfeed	0.00166	0.0000	0.01
KLBF	Kalbe Farma	0.00115	0.0011	0.01
LPPF	Matahari Department	0.00165	0.0003	0.01
LSIP	London Sumatra	0.00196	0.0000	0.01
MAPI	Mitra Adiperkasa	0.0048	0.4295	0.01
MIKA	Mitra Keluarga	0.00079	0.2027	0.01
MYOR	Mayora Indah	0.00074	0.0000	0.01
PGAS	Perusahaan Gas Negara	0.00234	0.0000	0.01
PTBA	Bukit Asam	0.0046	0.0000	0.01
PWON	Pakuwon Jati	0.00064	0.0009	0.01
SMRA	Summarecon Agung	0.00177	0.0000	0.01
TLKM	Telkom Indonesia	0.00171	0.1721	0.01
TPIA	Chandra Asri Pasific	0.00257	0.0000	0.01
UNTR	United Tractors	0.00389	0.0000	0.01

The Jarque Bera test results indicate that all assets were normally distributed at a significance level of 5%. Additionally, the results of the Augmented Dickey-Fuller (ADF) test in column 6 reject the null hypothesis of a unit root at a 5 % significance level, confirming that all return series were stationary. Leading to the identification of five stocks, namely ASII, INDF, MAPI, MIKA, and TLKM, which satisfied the criteria for both normal distribution and stationarity. All these stocks exhibited a standard deviation within the range of 2.7 % to 5.46 %. Among them, MAPI displayed the highest risk, while INDF had the lowest risk. Additionally, MAPI boasted the highest mean return at 0.48 %, while MIKA had the lowest mean return at 0.078 %. Notably, INDF demonstrated superior performance with a mean return of 0.124 %, indicating the best balance of return and risk compared to the other stocks. Detailed summary statistics for the selected stocks are presented in Table 2.

Table 2. Descriptive Statistics

Stock	Company	Sector	Mean	Std. Dev
ASII	Astra International	Misc. Industries	0.001979	0.03567

INDF	Indofood Sukses Makmur	Consumer Non-Cyclicals	0.001245	0.02724
MAPI	Mitra Adiperkasa	Non-Primary Consumer Goods	0.004805	0.05460
TLKM	Telkom Indonesia	Infrastructure	0.001709	0.03307
MIKA	Mitra Keluarga Karyasehat	Healthcare	0.000789	0.04459

The other data required in this study is the market return represented by the Jakarta Composite Index (JCI). JCI data is taken from Yahoo Finance. The expected market return of 0.194 % with a standard deviation of 1.781 % is obtained based on the period June 2020 - November 2024. While the average weekly sukuk yield is 0.1101 %, and the risk-free return is 0.0839 %. Figures 1 and 2 illustrate the trends in selected stock returns and prices:

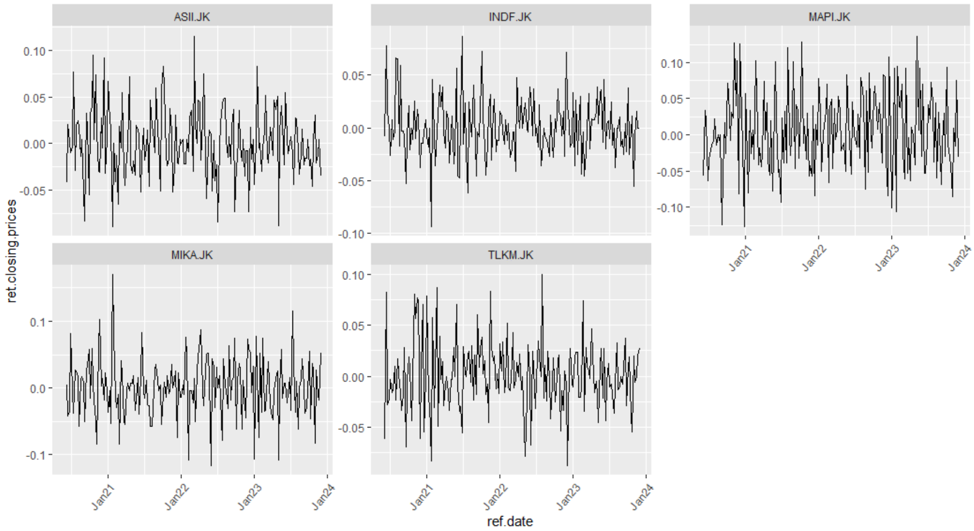

Figure 1. Plot of Stock Returns.

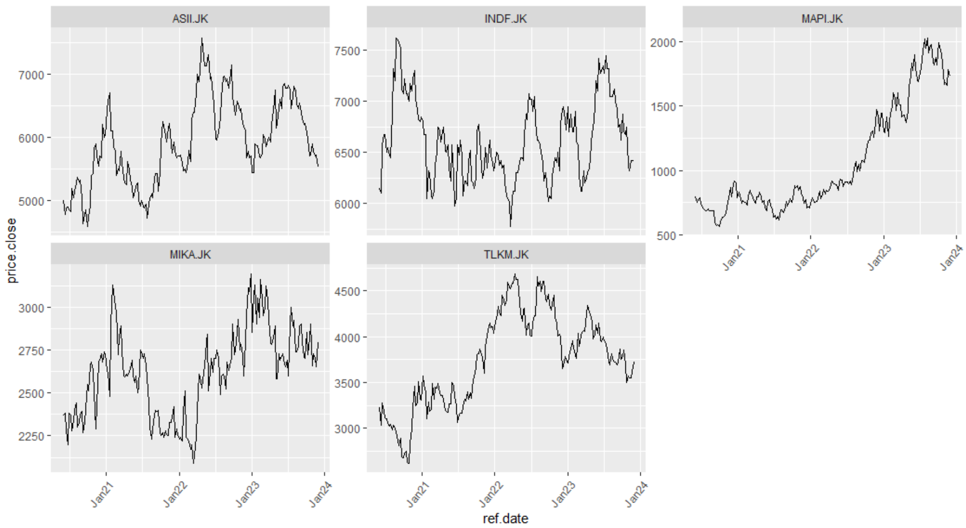

Figure 2. Plot of Stock Price.

Additionally, the objective is to identify the ideal allocation for each model created, including BCAPM, SCAPM, and Best Beta SCAPM. The portfolio formation results are subsequently computed to determine the most optimal model.

3.2. Model Development

This study integrates zakat and purification into the BCAPM framework to construct Shariah-compliant stock portfolios. The BCAPM is adapted to accommodate Shariah compliant stock portfolios, and its modified procedure is emphasized. The SCAPM return serves as a benchmark for market asset returns. Building on the insights of [14] and [15], the CAPM can be related to a regression model. Therefore, we assume that the SCAPM is approached through regression analysis, and it can be expressed in regression form as follows:

$$R_{it} - \left[\frac{R_{st}}{1-\delta_i}\right] = \alpha_i + \beta_i\left(R_{Mt} - \frac{R_{st}}{1-\delta_M}\right) + \varepsilon_{it} \tag{28}$$

We possess a market series (R_{Mt}), along with the return of each asset (R_{it}) in the actual dataset. The sukuk rate, R_{st}, may represent a constant profit rate. β_i denotes the beta of stock i, δ_M stands for the market purification rate, and ε_{it} represents the corresponding error term. We define a novel equilibrium excess return for Shariah-compliant stock as π_s, assuming ε_π is normally distributed.

In the context of the SCAPM analogous to the CAPM, the hypothesis $H_0 : \alpha = 0$ asserts the validity of the SCAPM. Estimating returns for the BCAPM model, incorporating adjustments for Shariah-compliant assets, follows a procedure akin to that of the original BCAPM. Moreover, given that SCAPM can be formulated as a regression equation, the optimization of the beta in SCAPM can be accomplished by utilizing the optimal beta, denoted as the best beta SCAPM:

$$\mu_i = \left[\frac{1}{1-\delta_i}\right]R_s + \beta_i^B\left[\mu_M - \frac{R_s}{1-\delta_M}\right] \tag{29}$$

In the BSCAPM, portfolio allocation is determined by minimizing risk, measured as the variance of returns. The constraints on the desired return are established based on investor preferences. The total allocation must sum up to 100 %, and short selling is not allowed, requiring all weights to be greater than 0.

$$\begin{aligned} \min \quad & w^T \Sigma_s w \text{s.t} \\ \text{s.t} \quad & w^T \hat{\mu}_s = \mu_{ps} \\ & w^T 1 = 1 \\ \text{and} \quad & w \geq 0 \end{aligned}$$

By using the Lagrange function, the weighting equation for BCAPM is obtained:

$$w_i = \frac{\Sigma_s^{-1}\mu_s}{\mu_s \Sigma_s^{-1} 1_{ps}^T} \tag{30}$$

where w_i denotes the vector of portfolio weights allocated to each asset.

3.3. Method

To implement the revised SCAPM incorporating sukuk, zakat, and purification, adjustments are made to both the mean and variance, considering zakat and purification requirements [16]. Purification for each asset is defined as the proportion of non-halal income relative to total income. As per DSN MUI guidelines, the maximum allowable ratio of non-halal to total income is less than 10 %. Access to the annual financial statements of each asset is available at https://www.idx.co.id.

Sukuk also provides a secure investment option for investors, guaranteed to be halal by DSN-MUI. As such, the sukuk rate is utilized as a substitute for the risk-free rate [17]. In Indonesia, sukuk types include Sukuk *Ijarah* (lease-based sukuk), Sukuk *Mudharobah* (profit-sharing sukuk), Sukuk *Musyarakah (*joint-venture sukuk), and Sukuk *Istishna* (project-based sukuk), with government sukuk categorized as Saving Sukuk and Retail Sukuk. Retail sukuk adheres strictly to Shariah principles, generating returns from investment profits. The sukuk rate demonstrated significant year-on-year increases from 2020 to 2024. In terms of liquidity risk, sukuk typically maintains high liquidity shortly after issuance, though this can diminish if it ranks lower [18]. Credit ratings of sukuk may be downgraded or face sporadic issuance from certain issuers [19].

The optimal beta in the model is derived by refining the beta in BCAPM. This optimal beta is believed to minimize pricing errors [1], enabling the SCAPM model, which incorporates adjustments for sharia principles, to create a more efficient portfolio. Additionally, to evaluate the performance differences between BCAPM and BSCAPM, the Sharpe ratio is used. This measure shows that BSCAPM performs better than BCAPM.

4. Result and Discussion

4.1. BCAPM Calculation

We outline the computation of BCAPM, with the outcomes of beta BCAPM presented in Table 3. The significance test details for alpha and beta are available in Columns 5 and 6 of Table 3.

Table 3. Alpha, beta, and the excess return BCAPM.

Stock	Alpha	Beta	*p-value*(Alpha)	*p-value*(Beta)	BCAPM
ASII	- 0.00923	0.99672	0.9786	8.52e−13	0.002043
INDF	- 0.00923	0.44045	0.8588	8.79e−05	0.00136
MAPI	- 0.00923	0.8496	0.4307	2.19e−04	0.001862
MIKA	- 0.00923	- 0.1506	0.7387	3.98e−02	0.000647
TLK M	- 0.00923	0.9751	0.8902	2.39e−14	0.002016

*Rejection of the null hypothesis at the 1 % significance level.

The validity of the BCAPM model for the five chosen stocks is confirmed as each p-value ≥ 0.05 satisfies the criteria. Additionally, the significance of the relationship between the return of each asset and the market return is established, as indicated by the p-value ≤ 0.05. The information presented in the table indicates that each asset's optimal beta is less than 1, signifying a low-risk stock. This implies that during a recession or economic downturn, each stock is generally stable concerning dividends, earnings, and overall market performance. Based on the expected returns, ASII is the top-performing stock with a return rate of 0.2043 %, surpassing all others. In contrast, MIKA stock offers the lowest return at 0.0647 %.

4.2. BCAPM Calculation

The inclusion of zakat and purification variables in the SCAPM model leads to a reduced anticipated portfolio return compared to the CAPM model. This is a direct consequence of the underlying assumption that the expected return diminishes and the variance contracts. Detailed information for purifying each asset is available in the appendices of respective company financial statements. As per the 2024 financial statements, the purification rate is determined based on the provided data.

$$\delta_i = \begin{pmatrix} 0.00934 & 0.01201 & 0.00225 & 0.01509 & 0.00596 \end{pmatrix}$$

In the context of SCAPM, we compute the purification differential for each asset by referencing the annual financial report. In Indonesia, a prerequisite for classifying a stock into Shariah-compliant groups

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is that its purification level remains below 10 %. Accounting professionals play a crucial role in determining the specific purification level of assets based on annual financial statements. The anticipated return of SCAPM, as depicted in Table 4.

Table 4. Beta Estimation, purification and return SCAPM

Stock	δ_i	$\frac{R_s}{1-\delta_i}$	Beta	SCAPM
ASII	0.00934	0.001361094	0.96085	0.002022
INDF	0.01201	0.001364772	0.42188	0.001656
MAPI	0.00225	0.001351422	0.79860	0.001902
MIKA	0.01509	0.001369040	- 0.15358	0.001264
TLKM	0.00166	0.001356466	0.93588	0.002000

Table 4 presents the estimated expected returns of the SCAPM model. Appears nearly identical to the BCAPM value found in Table 3. Despite being distinct, the disparity between the expected return values of SCAPM and BCAPM is minimal. As indicated in the SCAPM study conducted by [7], this similarity can be attributed to the adjustment for zakat and purification. Similar to SCAPM, estimating the expected excess return of Best Beta SCAPM involves considering several values, including the amount of zakat, sukuk yield, stock purification rate, and market. Additionally, the beta calculation utilizes equation (7) to achieve more optimal results. The calculation results with varying beta values are presented in Table 5 below:

Table 5. Estimation Best Beta, purification and return BSCAPM

Stock	δ_i	$\frac{R_s}{1-\delta_i}$	Best Beta	BSCAPM
ASII	0.00934	0.001361	0.99622	0.002045
INDF	0.01201	0.001365	0.44018	0.001667
MAPI	0.00225	0.001351	0.84857	0.001934
MIKA	0.01509	0.001369	- 0.15082	0.001265
TLKM	0.00166	0.001356	0.97462	0.002025

4.3. Allocation of Assets Within the Portfolio

In the preceding phase, we acquired estimates for the anticipated returns of BCAPM, SCAPM, and BSCAPM. Now, we will determine the allocation of each stock by applying the BCAPM method through Equation (20) and utilizing Equation (30) for BSCAPM. The process of optimizing weights through SCAPM is analogous to adjusting inputs in the mean-variance method, aiming for enhanced diversification outcomes. Furthermore, the beta employed in this approach represents the optimal beta. The outcomes obtained by applying the BCAPM, SCAPM, and BSCAPM approaches appear nearly identical upon initial observation, as illustrated in the provided Table 6 and Figure 3.

Table 6. Portfolio Weights for Each Method

Stock	BCAPM	SCAPM	BSCAPM
ASII	0.2601	0.2232	0.2237

INDF	0.2705	0.3176	0.3164
MAPI	0.0295	0.016	0.0171
MIKA	0.1216	0.1691	0.1681
TLKM	0.3182	0.2742	0.2747

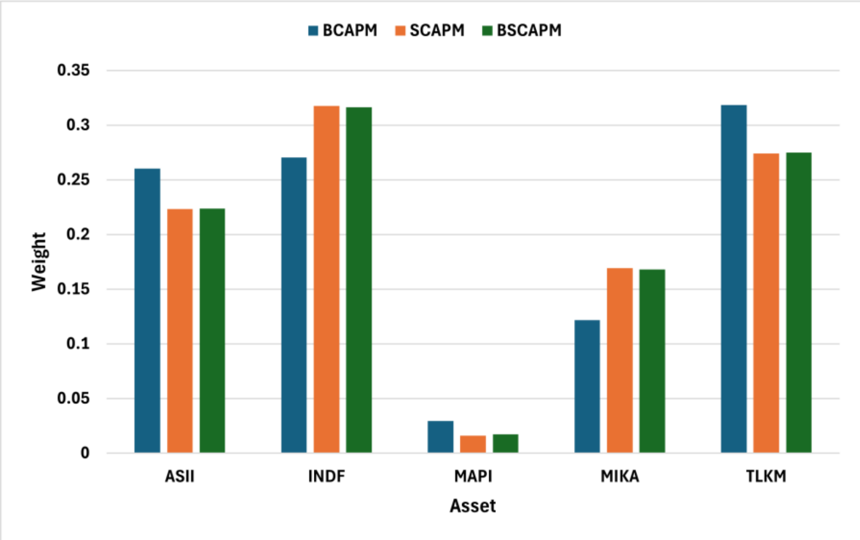

Figure 3. Weight of all portfolios

Figure 3 shows adjustments in the portfolio weights across several stocks. MAPI still receives the smallest weight in the portfolio, both with the BCAPM and BSCAPM methods. The second correction occurred in TLKM shares, which, in the BCAPM model, became the stock with the largest proportion; however, in the BSCAPM model, INDF shares had the largest weight. ASII and MAPI experienced the opposite correction, with a smaller weight for BSCAPM compared to the weight on BCAPM. Positive corrections are applied to INDF, MIKA, and TLKM stocks, resulting in larger weights than before. These changes reflect adjustments introduced during the second-stage optimization in the BSCAPM process. The results of the weights on each asset do not contain negative weights, thus meeting the constraint without short selling. This is particularly important for Muslim investors adhering to Sharia principles.

In this segment, we will present the outcomes of comparing two portfolios, wherein BCAPM is regarded as the traditional model, while BSCAPM is introduced as a novel alternative approach for allocating Islamic stocks. The weight outcomes derived from BSCAPM in Table 6 closely resemble those obtained through BCAPM. BSCAPM's advantage lies in its optimized beta, minimizing pricing errors and avoiding elements conflicting with Sharia principles during portfolio target return calculation. This approach sidesteps practices like interest usage, considers the impact of zakat on returns, and incorporates purification into a modelling framework aligned with Sharia principles. A comparison of the return, risk, and Sharpe ratio for the portfolios is presented in ratio Table 7.

Table 7. Portfolio performance results of BCAPM, SCAPM, and BSCAPM

Portfolio	Expected Return	Risk	Sharpe Ratio
ASII	0.2601	0.2232	0.2237
INDF	0.2705	0.3176	0.3164

It seems that the portfolio's return and risk are nearly identical when employing BSCAPM compared to BCAPM. The BSCAPM portfolio carries a slightly lower risk than BCAPM, potentially leading to an enhanced Sharpe performance. Specifically, the Sharpe ratio for BSCAPM stands at 5.35 %, surpassing the 4.38 % of BCAPM. In contrast to the BCAPM model, the portfolio formed using BSCAPM demonstrates a superior Sharpe ratio coupled with reduced risk.

Both models' Efficiency Frontier graphs are simultaneously displayed below to illustrate the efficient allocation outcomes.

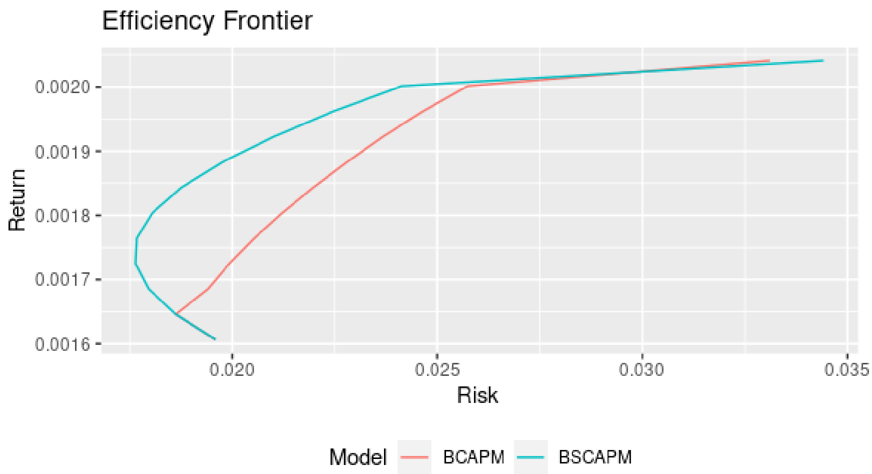

Figure 4. Efficiency Frontier Graphs of BCAPM and BSCAPM

Figure 4 illustrates that the efficiency frontiers of BSCAPM and BCAPM align within the risk range of 0.026 to 0.030. The potential risk and return outcomes of BSCAPM allocation closely resemble those of BCAPM. When facing risks surpassing 0.026, the BSCAPM allocation decision seems to produce portfolio returns exceeding those of BCAPM. Essentially, it suggests that at equivalent risk levels, the weights generated by BSCAPM lead to higher returns than BCAPM.

The portfolio constructed using the BSCAPM achieves a higher Sharpe ratio than that derived from the BCAPM. When evaluating the Shariah-compliant version of the BCAPM against its conventional counterpart, it was observed that the BCAPM, incorporating zakat and purification, outperformed the standard BCAPM. The inclusion of purification and zakat in the model leads to a reduction in the modified expected return and variance for each asset. Consequently, the Sharpe ratio of the portfolio increases. This discovery highlights that, despite altering the BCAPM benchmark, it yields an improved portfolio in terms of asset diversification and overall performance. Both the expected return and portfolio risks undergo changes in transitioning from the traditional to the Shariah version, marked by zakat reduction and purification, thereby introducing a novel Shariah-compliant strategy.

5. Conclusion

Lately, there has been a growing confidence in the global Islamic economic and financial system, aligning with advancements in pertinent research and the field of Shariah finance in investments. However, the available literature on Shariah modelling for portfolios remains inadequate. Integrating the Shariah framework into investment practices aims to accommodate the needs of Muslim investors. The study contributes to the underexplored domain of portfolio selection modeling within Islamic finance. The concept of Shariah principles appears to face some resistance. Numerous new terms are not well comprehended, and one such term is *riba*, referring to usury, which is strictly prohibited in Islam, including practices like bank interest. Other related terms include zakat from income and purification in stock markets following Shariah principles.

This study concentrates on two key aspects. Firstly, it seeks to enhance the BCAPM method by optimizing the beta in CAPM to minimize pricing errors. Additionally, the research puts forth a modification to the CAPM model, replacing the risk-free interest rate with the zakat factor, incorporating purification, and eliminating short sales. These dual enhancements lead to the creation of a novel optimal portfolio formation method known as BSCAPM.

The application of the BSCAPM, zakat, and purification models in the Indonesian stock market reveals a corrective adjustment in the weight of the BCAPM model. The portfolio return achieved through the BSCAPM model surpasses that of the BCAPM. Furthermore, performance evaluation using the Sharpe ratio indicates that BSCAPM outperforms BCAPM. These empirical findings indicate that, despite adjustments for zakat and purification, the BSCAPM continues to produce superior portfolio outcomes in terms of asset diversification, expected return, and portfolio risk. This study aims to provide valuable insights for investors, particularly those adhering to Islamic principles, to consider before engaging in investments in Islamic stocks.

6. Recommendation

Limitations of this study suggest potential avenues for future research. Future research should evaluate the global applicability of the BSCAPM model, particularly in markets with differing economic and regulatory environments. Furthermore, conducting sensitivity analyses could provide insights into how variations in crucial parameters, such as sukuk rates and purification criteria, influence the outcomes of the BSCAPM model. Developing a dynamic version of the model that accounts for variations in zakat rates and purification parameters over time could enhance its practical relevance and alignment with evolving Islamic finance principles. These enhancements would contribute to a more comprehensive understanding and application of the BSCAPM model across various contexts.

In future research, it would be valuable to assess the global applicability of the BSCAPM model, considering its adaptability to diverse international markets with varying economic conditions and regulatory frameworks. Additionally, sensitivity analyses could be conducted to understand how variations in key parameters, such as zakat rates and purification criteria, impact the outcomes of the BSCAPM model. A dynamic version of the model could be developed to account for changes in zakat rates and purification criteria over time, providing a more realistic representation of Islamic finance principles.

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