



## R Package Development for Difference Benchmarking in Small Area Estimation Fay-Herriot Model.

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**Abstract.** In recent decades, the use of small area estimation (SAE) for producing official statistics has been widely recognized by many National Statistics Offices including BPS-Statistics Indonesia. For official statistics usage, the aggregation of small area estimates is expected to be numerically consistent and more efficient than the aggregation of the unbiased direct estimates that cannot be guaranteed by Fay-Herriot model. Simulation experiments are performed to assess the behaviour of the difference benchmarking method Fay-Herriot model and to compare the mean squared error (MSE). The result shows that the difference benchmarking method can produce a consistent aggregation towards the direct estimation. Furthermore, an R package was built to implement the method that is easier to be used and is already available on the CRAN website. The package has been evaluated using validity (simulation), performance, case studies, and usability tests. These evaluations show that the package is suitable for use. Implementation of the methodology is also be applied to estimate average household consumption per capita expenditure in districts in D.I. Yogyakarta province, Indonesia 2019.

### 1. Introduction

Statistics Indonesia (BPS) has the responsibility of providing official statistics in Indonesia. In carrying out its responsibilities, BPS holds two main activities those are census and survey. Surveys are designed for obtaining reliable direct estimates in areas according to its design. However, direct estimates for small are based on sample data from surveys, are likely inefficient, and have large standard errors because the sample size is inadequate. Small area estimation (SAE) can improve the effectiveness of the sample size of surveys by borrowing the strength of neighboring areas and the relation between the set of auxiliary variables and the variable of interest [1]

However, Tzadivis et. al (2018) [2] stated that the consequence of using the Fay-Herriot model is when the estimates of small areas are aggregated, it was not necessarily consistent with the unbiased direct estimates in the larger area. While for official statistics use, aggregation of small area estimates is expected to be numerically consistent and more efficient than the aggregation of the unbiased direct estimates. Pfefferman and Barnard (2019) [3] proposed a benchmarking method called robust predictor. It is a modification to the optimal predictors that secures their robustness. The modification guarantees that the aggregate weighted mean of the small area predictors equals the corresponding weighted mean of the direct estimates. Hence, it is necessary to apply benchmarking method in the Fay-Herriot model to estimate official statistics indicators.



The difference benchmarking method concept will be applied in this paper, Rao and Molina (2015) [1] stated that difference benchmarking is one of a simple but widely used modifications to the empirical best linear unbiased predictions (EBLUP) by adding the EBLUPs estimates by the common adjustment factor. Difference benchmarking method never been applied in a software or statistics tools, when researcher want to apply difference benchmarking method, they need to create a complex syntax to run the method. This is certainly not efficient. Therefore, an R package was built to implement the method to be used easier. The R package that has been built will be used to do simulation and applied to estimate household consumption per capita expenditure in districts in D.I. Yogyakarta province, Indonesia.

## 2. Methodology

### 2.1. Fay-Herriot Model

Fay-Herriot model was first initiated by [4] using the EBLUP approach to estimate log per capita income (PCI) in the US. Fay-Herriot Model in general linear mixed model written as:

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{Z}\mathbf{u} + \mathbf{e}, \tag{1}$$

Where  $\mathbf{u}$  and  $\mathbf{e}$  are vectors of area random effects and sampling error.  $\mathbf{u}$  and  $\mathbf{e}$  are independently and distributed with:

$$\mathbf{u} \sim (\mathbf{0}, \mathbf{G}), \quad \mathbf{e} \sim (\mathbf{0}, \mathbf{R}), \tag{2}$$

Where  $\mathbf{G} = \mathbf{I}_m \sigma_u^2$  and  $\mathbf{R} = \mathbf{I}_m \sigma_e^2$  which  $\mathbf{I}_m$  is a  $m \times m$  identity matrix. Then the covariance matrix of  $\mathbf{y}$  is  $\boldsymbol{\Omega} = \mathbf{Z}\mathbf{G}\mathbf{Z}^T + \mathbf{R}$

Fay-Herriot model is approached using the best linear unbiased prediction (BLUP) proposed by [5]. However, because the area random effect of the population  $\sigma_u^2$  is unknown, it is empirically estimated using fisher scoring algorithm of REML that lead it to empirical best linear unbiased prediction (EBLUP).

### 2.2. Empirical Best Linear Unbiased Prediction (EBLUP)

As mentioned previously, EBLUP is the development of BLUP that has estimate the area random effects of population which is unknown in practical application. EBLUP estimators are obtained with replacing the  $\sigma_u^2$  by an estimator  $\hat{\sigma}_u^2$  of REML. EBLUP estimator in matrix form can be written as:

$$\hat{\boldsymbol{\mu}}^H(\hat{\boldsymbol{\delta}}) = \mathbf{X}\hat{\boldsymbol{\beta}} + \mathbf{Z}\hat{\mathbf{G}}\mathbf{Z}^T\hat{\boldsymbol{\Omega}}^{-1}(\mathbf{y} - \mathbf{X}\hat{\boldsymbol{\beta}}), \tag{3}$$

Where  $\hat{\boldsymbol{\mu}}^H$  is EBLUP estimation,  $\mathbf{X}$  is auxiliary variable,  $\mathbf{y}$  is direct estimates and  $\hat{\boldsymbol{\beta}}$  is the best linear unbiased estimator (BLUE) of  $\boldsymbol{\beta}$  written as:

$$\hat{\boldsymbol{\beta}} = \hat{\boldsymbol{\beta}}(\hat{\boldsymbol{\delta}}) = (\mathbf{X}^T\hat{\boldsymbol{\Omega}}^{-1}\mathbf{X})^{-1}(\mathbf{X}^T\hat{\boldsymbol{\Omega}}^{-1}\mathbf{y}) \tag{4}$$

2.2.1. MSE of EBLUP. Mean squared error of EBLUP estimator of Fay-Herriot model written as:

$$\text{MSE}(\hat{\boldsymbol{\mu}}_i^H) = E(\hat{\boldsymbol{\mu}}_i^H - \boldsymbol{\mu}_i)^2 \approx \mathbf{g1}_i(\sigma_u^2) + \mathbf{g2}_i(\sigma_u^2) + \mathbf{g3}_i(\sigma_u^2) \tag{5}$$

Same as [6], the estimator of MSE of EBLUP is obtained by substituting the REML estimator of  $\sigma_u$  and can be written as:

$$\text{mse}(\hat{\boldsymbol{\mu}}_i^H) \approx \mathbf{g1}_i(\hat{\sigma}_u^2) + \mathbf{g2}_i(\hat{\sigma}_u^2) + 2\mathbf{g3}(\hat{\sigma}_u^2) \tag{6}$$

In matrix form  $\mathbf{g1}(\hat{\sigma}_u^2)$ ,  $\mathbf{g2}(\hat{\sigma}_u^2)$  and  $\mathbf{g3}(\hat{\sigma}_u^2)$  can be written as:



$$\begin{aligned}
 \mathbf{g1}(\hat{\sigma}_u^2) &= \text{diag}[\mathbf{\Gamma R}] \\
 \mathbf{g2}(\hat{\sigma}_u^2) &= \text{diag}[(\mathbf{I} - \mathbf{\Gamma})\mathbf{X}(\mathbf{X}^T\hat{\mathbf{\Omega}}^{-1}\mathbf{X})]\mathbf{X}^T(\mathbf{I} - \mathbf{\Gamma})] \\
 \mathbf{g3}(\hat{\sigma}_u^2) &= \text{diag}[\sum_{k=1}^q \sum_{l=1}^q \text{cov}(\hat{\sigma}_{u(k)}, \hat{\sigma}_{u(l)})\mathbf{\Gamma}_{(k)}\hat{\mathbf{\Omega}}\mathbf{\Gamma}_{(l)}^T]
 \end{aligned} \tag{7}$$

Where  $\mathbf{\Gamma} = \mathbf{Z}\hat{\mathbf{G}}\mathbf{Z}^T\hat{\mathbf{\Omega}}^{-1}$ ,  $\mathbf{\Gamma}_{(k)} = \partial\mathbf{\Gamma}(\partial\hat{\sigma}_u^2)$  and  $\text{cov}(\hat{\sigma}_{u(k)}, \hat{\sigma}_{u(l)}) = \mathfrak{S}_{kl}^{-1}(\hat{\sigma}_u^2)$  which is obtained by the information fisher of fisher scoring in REML.

### 2.3. Benchmarking in Small Area Estimation

Suppose that  $\mu_i$  is the  $i^{th}$  area means and  $\mathbf{W}_i = \frac{N_i}{N}$  is a known proportion of units in area  $m$ . For All area are sampled ( $\sum_{i=1}^m \mathbf{W}_i = 1$ ), so  $\mu_+ = \sum_{i=1}^m \mathbf{W}_i\mu_i$  is the aggregate mean. Assuming that a reliable direct estimator  $\hat{\mu}_+ = \sum_{i=1}^m \mathbf{W}_i\hat{\mu}_i$  of  $\mu_+$  is desirable to ensure that the estimators of the small area mean  $\mu_i$ , when aggregated, agrees with the reliable direct estimators  $\hat{\mu}_+$ . The EBLUP estimators,  $\hat{\mu} = \hat{\mu}_i^H$ , do not satisfy this ‘‘benchmarking’’ property. In fact [1]

$$\hat{\mu}_+ - \sum_{i=1}^m \mathbf{W}_i\hat{\mu}_i = \sum_{i=1}^m \mathbf{W}_i(1 - \hat{\gamma}_i)(\hat{\mu}_i - \mathbf{x}_i^T\hat{\boldsymbol{\beta}}) \neq 0 \tag{8}$$

Therefore, it is necessary to modify the EBLUP estimators  $\hat{\mu}_i^H$ ,  $i = 1, \dots, m$  to ensure benchmarking. Rao and Molina (2015) [1] explains that there is a simple but widely use a benchmarking method called difference benchmarking.

2.3.1. *Difference Benchmarking.* Difference benchmarking is obtained by adding each  $\hat{\mu}_i^H$  by the common adjustment factor  $\alpha = \sum_{i=1}^m \mathbf{W}_i\hat{\mu}_i - \sum_{i=1}^m \mathbf{W}_i\hat{\mu}_i^H$ , leading to difference benchmarking estimator [1]

$$\hat{\mu}_i^{DB} = \hat{\mu}_i^H + \sum_{i=1}^m \mathbf{W}_i\hat{\mu}_i - \sum_{i=1}^m \mathbf{W}_i\hat{\mu}_i^H \tag{9}$$

2.3.2. *MSE of Difference Benchmarking.* Mean squared error (MSE) of difference benchmarking is obtained by adding a common term  $\mathbf{g4}(\hat{\sigma}_u^2)$  to the usual  $MSE(\hat{\mu}_i^H)$  [7]

$$\text{MSE}(\hat{\mu}_i^{DB}) \approx \text{MSE}(\hat{\mu}_i^H) + \mathbf{g4}(\hat{\sigma}_u^2) \tag{10}$$

This common term  $\mathbf{g4}(\hat{\sigma}_u^2)$  is given by [7]

$$\mathbf{g4}(\hat{\sigma}_u^2) = \sum_{i=1}^m \mathbf{W}_i\mathbf{B}^2(\hat{\sigma}_u^2 + \sigma_{ei}) - \sum_{i=1}^m \sum_{j=1}^m \mathbf{W}_i\mathbf{W}_j(1 - \gamma_i)(1 - \gamma_j)\mathbf{h}_{ij} \tag{11}$$

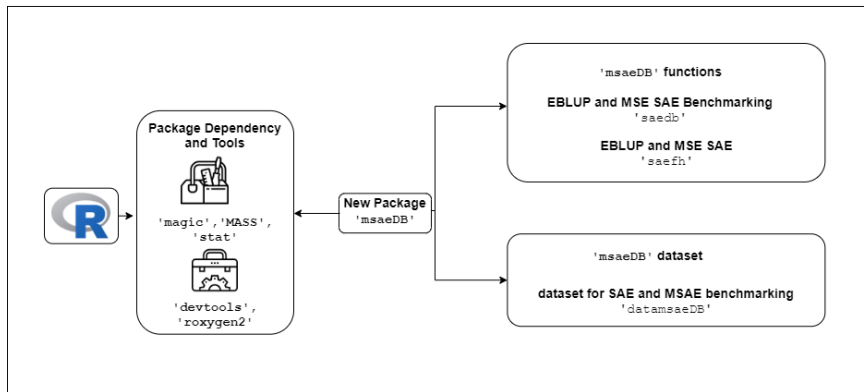
Where  $\mathbf{h}_{ij} = \mathbf{x}_i^T[\sum_{i=1}^m \mathbf{x}_i\mathbf{x}_j^T(\hat{\sigma}_u^2 + \sigma_{ei})^{-1}]^{-1}\mathbf{x}_j$  and  $\mathbf{B}_i = \sigma_{ei}(\hat{\sigma}_u^2 + \sigma_{ei})^{-1}$ . Steorts and Ghosh (2013) [7] has shown that the effect of benchmarking is to increase the MSE but very small relative to  $\text{mse}(\hat{\mu}_i^H)$  for all areas. Hence, in its application, difference benchmarking led to negligible inflation of MSE.

## 3. R Package Development

R Package was built by implementing difference benchmarking method to make it easier and accessible to everyone. R language was chosen because this is one of the opensource software and is widely used for data analysis.

### 3.1. The architecture of R Package

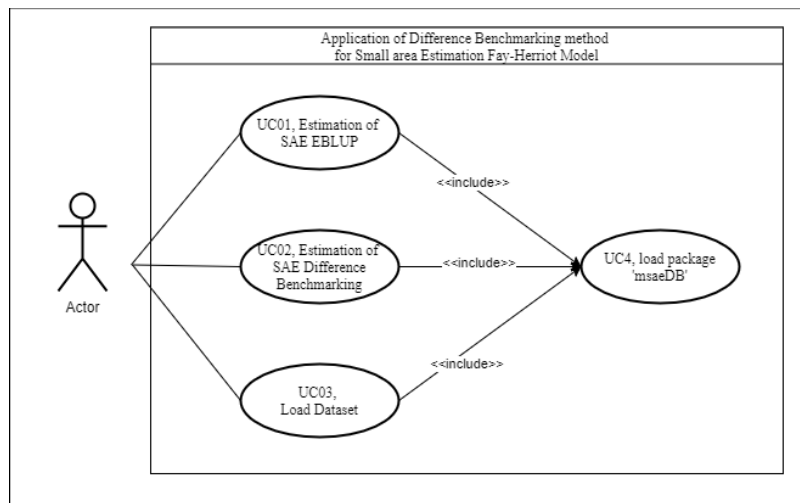
The R package to implement difference benchmarking for small area estimation was named ‘msaeDB’. To make the computational process easier, the package was built using a dependency package as tool in R package development. Package ‘msaeDB’ consists of some functions and datasets.



**Figure 1.** The architecture of Package ‘msaeDB’

**3.2. Use Case Diagram**

In package ‘msaeDB’, there is only an actor that will have interaction with the package in R software. The Actor can estimate EBLUP estimator and its MSE with or without difference benchmarking method. However, before starting to estimate, the actor must first load the package ‘msaeDB’. The interaction the actor toward the ‘msaeDB’ package was shown in Figure 2.



**Figure 2.** Use case diagram of package ‘msaeDB’

**3.3. Implementation**

**3.3.1. Implementation of Function Algorithm in R Package.** In package ‘msaeDB’ there are two functions. Function ‘saedb’ is used to estimate EBLUP with difference benchmarking and function ‘saefh’ to estimate EBLUP without difference benchmarking.

**3.3.2. Implementation of Dataset in R Package.** Package ‘msaeDB’ provides a dataset called ‘datamsaeDB’ to implement the ‘saedb’ and ‘saefh’ functions. This dataset consists of response variables, auxiliary variables, benchmarking weight, and variance, and covariance. Covariance is used when the actor is willing to simulate more than one response variables at a time using the multivariate Fay-Herriot model that will not be discussed in this paper.



3.3.3. *Documentation of ‘msaeDB’ in R Package.* Documentation of package ‘msaeDB’ is provided in <https://rdocumentation.org/packages/msaeDB/versions/0.2.1> consists of package description, list of functions and datasets, and the usage of the package

3.4. *Evaluation*

The package has been evaluated using validity (simulation), performance, case studies, and usability tests to make sure that the package is suitable and valid to be used. This subsection will only show the result of the performance and usability test. The validity and case study will be explained in the two next sections.

3.4.1. *Perform Test.* This test was carried to see the reliability of the program based on the computational time of each function. This test using the number of domains  $D = 100, 200, 800,$  and  $1000.$

**Table 1.** The computational time of functions based on number of areas

| Functions | Number of Areas |               |               |               |
|-----------|-----------------|---------------|---------------|---------------|
|           | 100             | 200           | 800           | 1000          |
| ‘saefh’   | 1.32 seconds    | 10.50 seconds | 13.92 minutes | 26.92 minutes |
| ‘saedb’   | 1.75 seconds    | 12.50 seconds | 16.31 minutes | 31.16 minutes |

Function ‘saedb’ takes time longer than ‘saefh’. Because, there is an additional process according to the benchmarking method.

3.4.2. *System Usability Scale (SUS).* This test was carried to see whether the package is acceptable by the user or not. This test was carried using the instrument of *System Usability Scale (SUS)* consists of ten questions against 14 respondents from STIS Polytechnic of Statistics and Statistics Indonesia (BPS). The respondents have firstly given the guide and brief explanation of the package and documentation about the usage of the package. The score of this test is 76,07 which means that the package ‘msaeDB’ is suitable and acceptable for use.

**4. Simulation**

The simulation was carried out to see whether the algorithm of the functions gives a valid and appropriate output of difference benchmarking Fay-Herriot model. Simulation data is generated by the following steps:

1. Generate  $x_d^b, u_d^{(b)}, e_d^{(b)}$  and  $w_d^b$ , where  $d = 1, \dots, D$  and  $b = 1, \dots, B.$
2. Calculate direct estimation  $\hat{\mu}^b$ , where  $y = X\beta + u_i + e_i.$
3. Calculate EBLUP Estimators ( $\hat{\mu}_i^H$ ).
4. Calculate difference benchmarking ( $\hat{\mu}_i^{DB}$ ).
5. Calculate MSE of each model.
6. Repeat steps 1 to 5  $B=100$  times for each step.

This simulation was using number of domains  $D = 30, 50, 100$  and  $200.$   $x_1 \sim Unif(10,1), x_2 \sim Unif(10,2), w \sim Unif(25,30)$  and  $\sigma_e = 0.15.$  Simulation result is shown in Table 2.

**Table 2.** The MSE of EBLUP with and without difference benchmarking

| Methods  | Number of Areas |          |          |          |
|----------|-----------------|----------|----------|----------|
|          | 30              | 50       | 100      | 200      |
| EBLUP    | 0.099360        | 0.091288 | 0.089988 | 0.086987 |
| EBLUP DB | 0.099365        | 0.091292 | 0.089990 | 0.086988 |



Table 2 shows that the MSEs of all estimators decreases as the number of domains D increases. Same as Steorts and Ghosh (2013), difference benchmarking gives effect to increase the MSE but very small relative to  $mse(\hat{\mu}^H)$  for all areas. Based on this simulation, can we conclude that the algorithm of the functions has given valid and appropriate output of difference benchmarking Fay-Herriot model.

**5. Case Study**

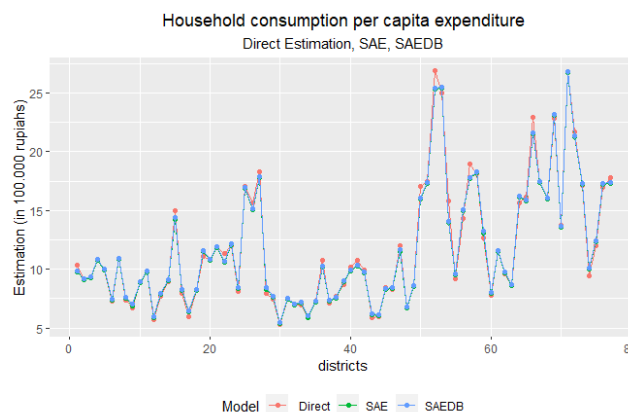
An R package that has been build is implemented to estimate average household consumption per capita expenditure (HCPE) data in Yogyakarta Province, Indonesia. HCPE data has been collected by the National Socioeconomic Survey (SUSENAS) 2019 which is regularly conducted by Statistics Indonesia.

For the corresponding area-specific auxiliary variables, there are ten variables that we use: number of junior high schools ( $x_1$ ), number of senior high schools ( $x_2$ ), number of vocational schools ( $x_3$ ), number of universities ( $x_4$ ), number of doctor practices ( $x_5$ ), number of restaurants ( $x_6$ ), number of hotels ( $x_7$ ), number of small shops ( $x_8$ ), number of private banks ( $x_9$ ), and number of village cooperation unit ( $x_{10}$ ). The ten auxiliary variables are obtained from Yogyakarta Village Potential 2019. The chosen auxiliary variables reference to [10] with some additional variable that is correspondent and significant to response variable based on stepwise method. The regression parameter estimates of HCPE are shown in Table 3.

**Table 3.** Regression parameter estimates of average HCPE

| Variables | Estimate | Std. Error | p-value |
|-----------|----------|------------|---------|
| Constant  | 110.6700 | 9.5295     | 0.0000  |
| $x_1$     | -4.0612  | 1.6762     | 0.0154  |
| $x_2$     | 5.8130   | 2.4051     | 0.0156  |
| $x_3$     | -7.2192  | 2.3944     | 0.0026  |
| $x_4$     | 2.3537   | 1.5332     | 0.1248  |
| $x_5$     | 2.3084   | 0.7402     | 0.0082  |
| $x_6$     | -0.5315  | 0.1987     | 0.0075  |
| $x_7$     | 1.1816   | 0.4262     | 0.0056  |
| $x_8$     | 0.0280   | 0.0127     | 0.0282  |
| $x_9$     | 3.0072   | 1.6446     | 0.0675  |
| $x_{10}$  | -5.1898  | 3.3469     | 0.1210  |

By using the number of populations of each district, then calculating the estimation using package ‘msaeDB’, the estimation results are obtained as shown in Figure 3.



**Figure 3.** Estimation of direct estimation, EBLUP and EBLUP without the benchmark



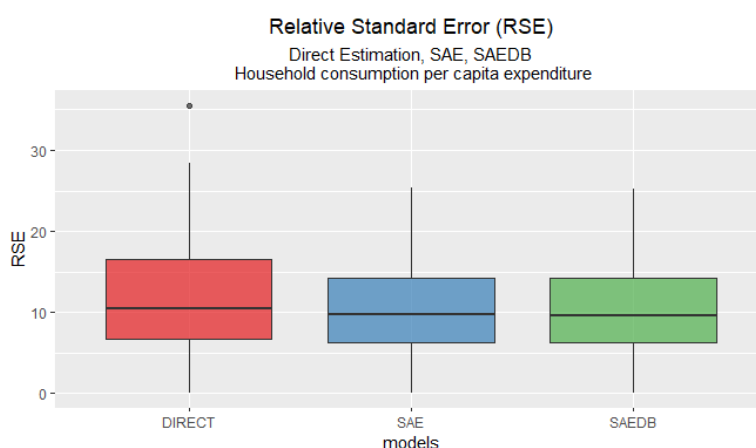
From Figure 3, we can see that estimation of EBLUP and EBLUP with difference benchmarking have given almost the same pattern, a district with a high value of average in one model HCPE will also have a high value in other models.

Table 4 shows the summary of estimation of direct estimation, EBLUP, and EBLUP with difference benchmarking. The estimation of direct estimates, EBLUP, and EBLUP with difference benchmarking have given almost the same values for each measurement of central tendency.

**Table 4.** Summary of Estimation of average HCPE (in 100.000 rupiahs)

| Summary             | Direct | EBLUP  | EBLUP DB |
|---------------------|--------|--------|----------|
| Minimum             | 5.332  | 5.404  | 5.492    |
| 1 <sup>st</sup> Qu. | 7.927  | 8.175  | 8.262    |
| Median              | 10.185 | 9.930  | 10.018   |
| Mean                | 11.844 | 11.756 | 11.843   |
| 3 <sup>rd</sup> Qu. | 15.653 | 15.080 | 15.167   |
| Maximum             | 26.865 | 26.715 | 26.803   |

Figure 4. shows the box plot of RSE of direct estimates, EBLUP estimates, and EBLUP with the difference benchmarking estimates based on the Fay-Herriot model. It shows that estimation using EBLUP and EBLUP with difference benchmarking can give a more efficient estimation based on the RSE.



**Figure 4.** RSE of direct estimation, EBLUP and EBLUP without benchmarking

The summary of each measurement of central tendency RSE of direct estimates, EBLUP, and EBLUP with difference benchmarking estimates are shown in Table 5.

**Table 5.** Summary of RSE of average HCPE in each model

| Summary             | Direct | EBLUP  | EBLUP DB |
|---------------------|--------|--------|----------|
| Minimum             | 0.000  | 0.000  | 0.004    |
| 1 <sup>st</sup> Qu. | 6.754  | 6.329  | 6.283    |
| Median              | 10.413 | 9.964  | 9.621    |
| Mean                | 11.698 | 10.057 | 9.984    |
| 3 <sup>rd</sup> Qu. | 16.611 | 14.262 | 14.171   |
| Maximum             | 35.554 | 25.431 | 25.282   |

By choosing a suitable benchmarking weight, the RSE of EBLUP with difference benchmarking generally will have more efficient RSE than direct estimates and EBLUP estimates. Table 6. shows the aggregation of direct estimates, EBLUP and EBLUP with difference benchmarking.



**Table 6.** Summary of RSE of average HCPE in each model

| Models           | Aggregation |
|------------------|-------------|
| Direct Estimates | 13.12367    |
| EBLUP DB         | 13.12367    |
| EBLUP            | 13.03612    |

From Table 6 we can see that EBLUP with difference benchmarking has given a consistent aggregation to its direct estimates, unlike the EBLUP estimates.

## 6. Conclusion

We have shown that EBLUP with difference benchmarking has given a consistent aggregation, unlike the EBLUP estimation. R package to implement difference benchmarking for Fay-Herriot model is successfully developed and can be accessible on <https://cran.r-project.org/package=msaeDB> and has a system usability score of 76.07 indicates that the package is acceptable by the user. Based on its computational times, the function that implements difference benchmarking gives a longer time because of the additional process of benchmarking. Simulation shows that the algorithm of the package has given a valid output. The implementation of difference benchmarking to estimate the average of household consumption per capita income HCPE in Yogyakarta Province Indonesia shows that EBLUP with difference benchmarking can give a consistent aggregation to its direct estimates, unlike the EBLUP without difference benchmarking.

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