



Extreme Value Theory: Modelling Catastrophic Losses In Sports Injury

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Abstract. Using Extreme Value Theory with a peaks-over-threshold method, we modelled the top 2% of sports-injury losses from 200,000 simulated claims. A generalized Pareto fit via MLE yielded a positive shape ($\xi = 0.783$), indicating a fat tail where rare injuries dominate severity. Q-Q and P-P diagnostics show good agreement between model and data. The implied 100-year loss is round 3.31 billion (currency units), and TVaR confirms that conditional on approaching the tail, predicted losses increase quickly. These findings support need for capital buffer to mitigate costly injuries, severe-scenario stress testing, and pricing loadings that specifically consider for costly but rare injuries.

Keyword: Extreme Value Theory, Generalized Pareto Distribution, Peaks Over Threshold,

1. Introduction

In professional and high-performance athletics, the risk of injury is an inherent and accepted part of training and competition. While sports medicine has made significant contribution in preventing and treating common injuries, a tough challenge remains in understanding and managing the risk of rare, severe events. These are injuries that are not only severe in their physical impact but also extreme in their financial consequences.

Sports injury are chosen instead of other domains such as finance or environment due to the financial risks associated with professional and high-performance athletics. Because extreme cases may cause severe physical impact & consequences, it is important to allocate capital in order to fund such claims when they occur, consider extreme scenarios in premium pricing, and incorporate stress testing which consider such cases. Hence, it is important to model these events accurately in order to be able to plan and execute the appropriate course of actions effectively.

Similar studies have used similar methodology. As an example, Adil & Huai (2023) used Extreme Value Theory using the block maxima method in order to model the highest possible earthquake magnitude in Makran subduction zone. Extreme parameters are fitted using the generalized extreme value distribution. The results showed that the maximum magnitude tend to increase in the next 100 years and the shape parameter equals 0.29. Another example is research by Daniel & Maashele (2023) about using Extreme Value Theory to model Johannesburg Stock Exchange Financial Market Data. The goal is to compare the block maxima approach & peaks-over-threshold (POT) approach. POT approach return level estimates were higher than the block maxima's return level approach. The study also showed that using blended generalized extreme value may be better for short-term forecasting. Unlike previous studies focused on finance and natural disasters, this paper focuses on modelling sports injuries.



The fundamental problem with these extreme events is their low frequency. It is exceptionally difficult, if not impossible, to build a reliable risk model using only historical data, as a typical dataset may contain few, if any, of these extreme events. This data scarcity leaves insurers and risk managers in a unreliable position; traditional actuarial methods, which rely on historical averages and predictable deviations, are unfit to price such risks or to quantify the true magnitude of a potential worst-case scenario. Without a robust statistical framework, putting adequate financial reserves and fair premiums becomes a matter of educated guesswork, increasing the risk of under-reserving and potential insolvency.

To address this critical gap, this research employs a powerful combination of stochastic simulation and Extreme Value Theory (EVT). By first simulating a large, simulated dataset that mathematically resembles the features of a heavy-tailed risk profile, we overcome the problems of limited historical data. Subsequently, we apply the Peaks-over-Threshold (POT) methodology, a foundation of EVT, to specifically model the tail of the loss distribution.

2. Research Method

This study employs a quantitative, model-based approach to analyze the financial impact of extreme athletic injuries. The methodology is structured in two primary stages: (1) Data Generation and Preparation, which creates a suitable dataset for analysis, and (2) Data Analysis, which applies a suite of techniques from Extreme Value Theory (EVT) to model risk and derive actionable insights. The entire process is implemented in Python, leveraging the `scipy`, `pandas`, and specialized `pyextremes` libraries.

2.1. Data Collection

A key challenge in modelling catastrophic risk is the limited amount of historical data for low-frequency, high-severity events. To overcome this limitation and to ensure a dataset with the theoretical properties for EVT, this research utilizes a stochastic simulation approach. Since the data is simulated, it does serve as a real-world data but acts as a method to prove that the idea is realistic.

As shown in Table 1, the primary dataset was synthetically generated by drawing 200,000 observations from a Pareto distribution using the `scipy.stats.pareto` module. The choice of the Pareto distribution is theoretically motivated; by the Pickands–Balkema–de Haan theorem, the distribution of exceedances over a sufficiently high threshold from a wide range of underlying distributions (including the Pareto) approximates to a Generalized Pareto Distribution (GPD). This makes a Pareto-generated dataset the ideal standard for validating an EVT model, as its tail behavior is known *a priori*.

The distribution was parameterized as follows:

- Shape (β): 1.2. A shape parameter less than 2 ensures a "heavy tail" with unlimited variance, accurately reflecting the nature of catastrophic losses where outlier events are possible and significant.
- Location (loc): 1,000. Establishes a minimum loss value.
- Scale (α): 50,000. Determines the spread and magnitude of the data to model realistic financial values.

From the simulated dataset, a threshold-based selection procedure was applied to extract exceedances. Following the peaks-over-threshold (POT) approach, the 98th percentile was chosen as the cutoff. This threshold balances bias and variance in EVT analysis: a lower threshold may introduce bias by including too many non-extreme points, while a higher threshold may leave too few observations for reliable estimation. The threshold was 1.29 million and produced 2,454 extreme events, which were used as the main data for fitting the generalized Pareto distribution (GPD) as shown in table 1. This step models the process of filtering operational, insurance, or financial loss data where only the extreme events are used for risk calculation.

**Table 1.** Dataset summary and threshold selection

Statistic	Value
Total Simulated Data	200,000
Threshold (98 th percentile)	1.29 million
Number of Exceedances	2,454

2.2. Data Analysis Techniques

The analytical phase employed Extreme Value Theory (EVT), focusing on the GPD under the POT framework. The selected exceedances were modelled using the maximum likelihood estimation (MLE) method to estimate the shape parameter (ξ), which indicates tail heaviness, and the scale parameter (β), which shows the distribution of extreme values. A positive ξ , as found in this study, confirms the presence of fat tails and highlights the non-negligible probability of catastrophic outcomes.

Model validation was carried out via diagnostic plots, including return value plots, probability density plots, Q-Q plots, and P-P plots. These visual diagnostic tools test whether the GPD adequately shows the observed tail data. In particular, the Q-Q and P-P plots assess alignment between observed and theoretical quantiles, while the return value plot provides practical benchmarks for return period estimates.

Subsequently, return level estimation was done to measure expected losses for different return time horizons (e.g., 10-year, 50-year, 100-year events). Additional analyses included conditional exceedance probabilities, which evaluate escalation risk once a threshold is violated, and expected shortfall (TVaR), a more traditional tail risk measure that considers losses beyond the VaR cutoff. Finally, simulation of new scenarios from allowed stress testing and further model validation. To ensure that the research is accurate and reliable, robustness checks will be conducted. This includes different thresholds, namely 90%, 95% and 99%, followed by using different number of observations, namely 50000, 100000, and 200000 observations as well as using block maxima method instead.

Robustness checks are done by estimating the severity of extreme injuries via two methods. The first method is by using Peaks-over-Threshold. First, choose a high threshold u with 90%, 95%, and 99% as the chosen quantiles. Then, decluster exceedances to avoid serial dependence via run-length windows, $r = \{12H, 24H, 48H\}$ and keep the top of the cluster only. Next, estimate GPD parameters, (ξ, σ) , on exceedances, $Y = X-u > 0$. The primary method of estimation is maximum likelihood estimator using SciPy library in Python but if tail sample is too thin then use Hill's tail index and a log-log survival slope. Afterwards, estimate the exceedances rate, λ and compute the return level as follows:

$$RL_t = u + \frac{\sigma}{\xi} \{ (T\lambda)^{\xi-1} \}, \xi \neq 0; RL_t \approx u + \sigma \log(T\lambda), \xi \approx 0 \quad (1)$$

For the block maxima method, aggregate maxima over fixed blocks (365D, 180D, 90D, 30D) then fit Generalized Extreme Value (GEV) distribution via pyextreme library in Python and read the return level, RL_t , directly at $T = 100$ years. The explanation behind this methodology is POT uses more tail data but depends on threshold. Block maxima is independent of threshold but depends on block choice and can require lots of data. Before performing any comparison, it is crucial to ensure the tail is well-poised via 2 visual diagnostics, i.e mean life residual plot & threshold stability plot. For mean life residual plot, mean excess, $e(u) = E[X-u|X > U]$ is plotted against u . On the other hand, threshold stability plots visualizes $\xi(u)$ and $\sigma(u)$ traced across a grid of u then pick the lowest value u in which ξ, σ stabilize while maintaining an adequate amount of exceedances.

To be able to compare the robustness of the 2 methods, RL_{100} is recomputed across 4 dimensions as follows:



- $n \in \{50000, 100000, 200000\}$
- $q \in \{0.90, 0.95, 0.99\}$
- $r \in \{12H, 24H, 48H\}$
- Block Size $\in \{365D, 180D, 90D, 30D\}$

For each method, the absolute relative gap is computed as follows & if the gaps persist at 15% at larger sample size then there is sensitivity for method choice:

$$\text{Gap} = \frac{|RL_{100}^{\text{BM}} - RL_{100}^{\text{POT}}|}{RL_{100}^{\text{POT}}} \quad (2)$$

The limitations of using simulations in modelling real-world phenomenon is that the data may not be an exactly the same as the data in the real-world so the any analyses resulting from the data may not be accurate and reliable. This is because simulation uses assumptions which tend simplify reality so it may not fully reflect real-world phenomenon. Hence, the results may not be applicable if the assumptions do not approximate or reflect the real-world. Furthermore, the choice of distribution in the simulation used to generate synthetic data may yield different observations. Hence, the results and analyses might be different if a different distribution is used which makes it not robust and also again, this choice is based on assumptions hence may not fully reflect reality.

3. Result and Discussion

3.1. Threshold Selection and Extreme Value Extraction

The first step of analysis involved identifying the appropriate threshold for extreme value modelling. By applying the peaks-over-threshold (POT) method, the 98th percentile was chosen as the cutoff point, converging to a value of approximately 1.29 million. This ensured that the dataset covered to only only the most severe losses while maintained enough sample size. From the original 200,000 simulated observations, 2,454 exceedances were identified.

This result confirms the expected rarity of extreme losses under heavy-tailed distributions: less than 2% of total observations were classified as “extreme.” The distribution of exceedances indicated strong right-skewness, suggesting that extreme value theory is an appropriate framework for deeper analysis. Importantly, the threshold chosen balances model bias and variance, ensuring stability of estimates while obtaining enough extreme observations for statistical modelling.

3.2. Fitting the Generalized Pareto Distribution

Once the exceedances were identified, they were modeled using the generalized Pareto distribution (GPD). The maximum likelihood estimation (MLE) produced two key parameter estimates: a shape parameter (ξ) of 0.783 and a scale parameter (β) of approximately 1.81 million as summarized in the Table 2.

The positive value of ξ is a crucial finding. It indicates that the loss distribution is heavy-tailed, meaning the probability of observing very large events decreases slowly rather than sharply. In reality, this shows that very huge financial losses, although have very low frequency, cannot be ignored due to their disproportionately large financial impact. A light-tailed distribution ($\xi \leq 0$) would underestimate such risks, leading to insufficient reserves.

The fitted model was evaluated using log-likelihood ($-39,734.8$) and Akaike Information Criterion (AIC = 79,474), as shown in table 2. While absolute values of these statistics are less informative without comparison, they confirm that the GPD provides a stable and consistent result. More importantly, diagnostic validation demonstrates that the model aligns closely with observed extremes.

Table 2. Estimated parameters of the GPD model

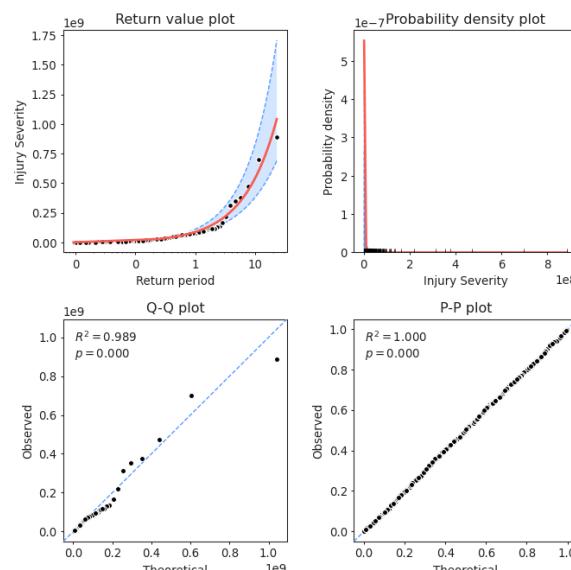
Parameter	Value
Shape	0.783
Scale	1.81 million
Log-likelihood	-39,734.8
AIC	79,474

3.3. Model Validation via Diagnostic Visualizations

A range of diagnostic tools were used to evaluate model fit. The return value plot showed a strong alignment between the fitted GPD and the observed data across most return periods, although the uncertainty interval widened significantly for long periods of time. This widening reflects the difficulty of forecasting very rare events with limited data.

The probability density plot proved the fat-tailed nature of the distribution, showing a rapid increase near the threshold and a long right tail. Both the Q-Q plot ($R^2 = 0.989$) and the P-P plot ($R^2 = 1.0$) demonstrated excellent fit, confirming that the GPD captured the quantile behaviour of observed extremes almost perfectly. Slight deviations appeared at the most extreme quantiles, but such deviations are due to the sample size.

These diagnostic results assure that the GPD is an appropriate model for the dataset, supporting its use for tail risk estimation, as summarized in figure 1.

**Figure 1.** Diagnostic plots

3.4. Estimation of Return Level

Return level estimation provides metrics for risk management. Results show that a 2-year event corresponds to a return level of approximately 154 million, a 10-year event to 545 million, and a 100-year event to 3.31 billion. A 100-year return level of around 3.31 billion suggests that insurers trying to cover elite athletes may face extreme losses hence sufficient reserves, appropriate pricing methods and reinsurance strategies need to be adjusted to consider these risk factors. Confidence intervals widened at greater time horizons, indicating greater uncertainty.



This highlights an important problem in extreme value analysis: while short-term return levels can be estimated with accuracy, long-term predictions (e.g., 100-year losses) have large confidence intervals. For actuaries, this highlights the importance of stress testing and simulation rather than relying only on point estimates. Nevertheless, the 100-year return level provides a benchmark for solvency planning, in accordance with regulatory requirements such as Solvency II and risk-based capital frameworks. Table 3 summarizes the return level estimates along with their respective confidence intervals.

Table 3. Return Level & Confidence Intervals

Return Periods (years)	Return Level Estimates	Lower Confidence Interval	Upper Confidence Interval
2	153963600	120082300	205568400
5	316560100	232791600	456168600
10	545418900	382297100	833148200
25	1118682000	735405300	1846639000
50	1925565000	1205363000	3371455000
100	3313902000	1975116000	6155226000

3.5. Conditional Exceedances Probabilities

Conditional exceedance analysis adds practical insight into escalation risks. For example, given that a loss already exceeds 50 million, the probability of it exceeding 200 million is approximately 16.7%. This result is particularly important for layered insurance and reinsurance, where the concern is not just whether a threshold is breached but how much further losses may escalate.

From a business perspective, this suggests that once moderate losses occur, there remains a significant chance of escalation to catastrophic levels. This reinforces the necessity of reinsurance programs and contingency planning to absorb unexpected shocks. Connecting to sports injury, if an elite athlete faces moderate losses then there is a high chance that it may escalate to extreme levels. Hence, insurers need to prepare sufficient reserves, set appropriate premium pricing, and set reinsurance strategies so that extreme financial losses can be mitigated.

3.6. Expected Shortfall (TVaR)

Expected shortfall (TVaR) was analyzed as a complementary risk measure to value-at-risk (VaR). The TVaR curve revealed that expected losses increase sharply as the confidence level approaches 100% as shown in figure 2. At extreme quantiles, TVaR exceeded 1 billion, far surpassing the VaR values at corresponding confidence levels.

This finding highlights the weaknesses of VaR as an individual risk measure. While VaR shows the threshold loss level at a given probability, it ignores the size of losses beyond that point. In contrast, TVaR accounts for these catastrophic outcomes, offering a more conservative and informative metric. This can more accurately model scenarios in which an athlete faces extreme financial losses. For actuarial practice, this makes TVaR particularly useful for setting solvency capital requirements and pricing reinsurance.

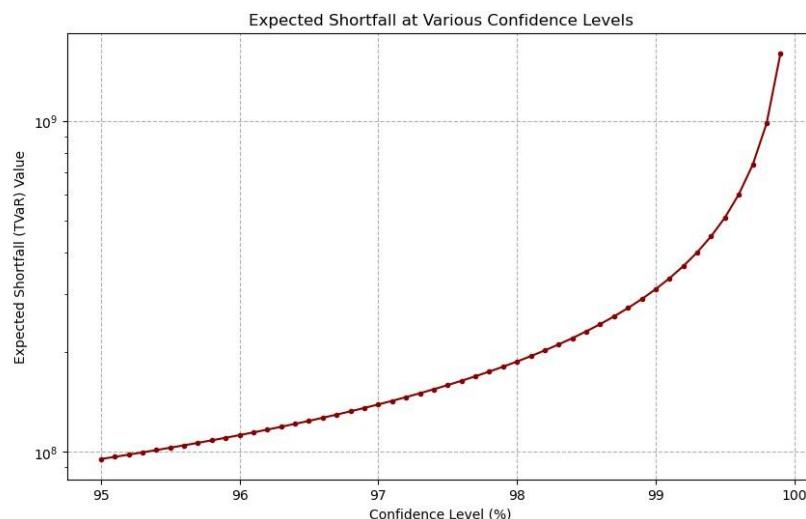


Figure 2. TVaR at various confidence levels

3.7. Contribution to Expected Shortfall

A decomposition of the 99% TVaR highlighted the concentration of risk in the largest events. Losses between 86 million and 240 million contributed less than 30% of the tail risk, while losses beyond 240 million contributed over 60%. This finding shows that a small number of events rule the overall risk profile. The findings are summarized in table 4:

Table 4. Contribution to expected shortfall

Loss Intervals	Contribution to Total Risk in Tail
[86307241, 106188613]	7.70%
[106188613, 141714367]	9.92%
[141714367, 240645918]	14.70%
[240645918, 10729359378]	60.77%

This insight has direct implications for insurance and reinsurance design. It suggests that moderate risk layers contribute relatively little to overall tail risk, while catastrophic layers rule the exposure. As a result, reinsurance contracts and capital reserves should focus heavily on protecting against extreme levels of risk. Connecting to sport injury, catastrophic injuries may rule the amount of financial losses so appropriate actions should be taken to mitigate against these extreme losses.

3.8. Statistical Moments of the Tail

The tail distribution exhibited extreme skewness (84.07) and kurtosis (9,311). These values far exceed those of normal or even moderately skewed distributions, confirming that the dataset is ruled by rare, very large outliers. The high kurtosis in particular reflects the “fat-tailed” nature of the distribution, where the probability of extreme outcomes is significantly higher than under conventional models, as summarized in table 5.

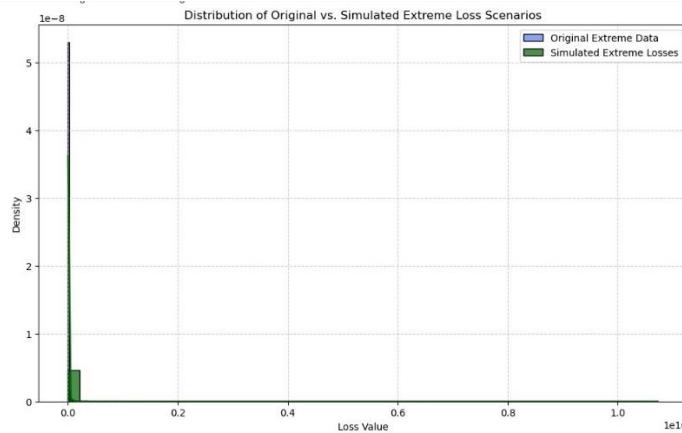
This result aligns with theoretical properties of heavy-tailed risk and confirms the application of extreme value theory. Ignoring such characteristics would lead to severe underestimation of capital & solvency requirements. Relating to sport injuries, there is a high chance that moderate losses will increase to extreme losses, proven by the high and positive skewness parameter (84.07) and the amount of extreme losses will be very high as shows by the very high kurtosis (9311.08).

**Table 5.** Statistical moments of tail

Parameter	Value
Skewness	84.07
Kurtosis	9311.08

3.9. Simulation of New Extreme Scenarios

To test the predictive capacity of the fitted model, 50,000 new extreme loss scenarios were simulated from the GPD. The resulting distribution closely resembled the original exceedances, indicating that the GPD captures the underlying tail dynamics, as shown below in figure 3. These kinds of simulations are important for stress testing and risk scenario planning, as they provide insight into potential future catastrophic events. From an actuarial standpoint, this step provides practical methods for reserving, pricing, and loss quantification.

**Figure 3.** Histogram/density plot of simulated vs observed exceedances

3.10. Robustness Checks

The diagnostics support a GPD tail for severe sports-injury costs. The mean residual life curve is approximately linear across the full threshold grid, and the threshold-stability across full threshold grid, and the threshold-stability plot shows ξ stabilizing around 0.80 with a smooth σ once the threshold approaches the 1000000 up until 2000000 range; in this the number of exceedances is around 2830 up to 3091. The threshold is set at $u \approx 1.0 \times 10^6$ and the declustering window is 24 hours.

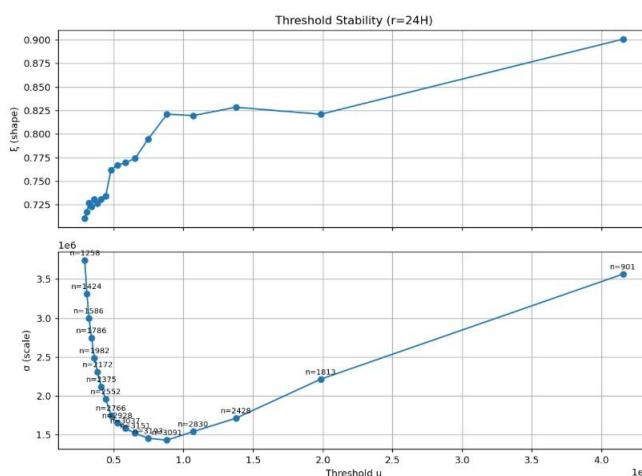




Figure 4. Threshold stability plot
Mean Residual Life (MRL)

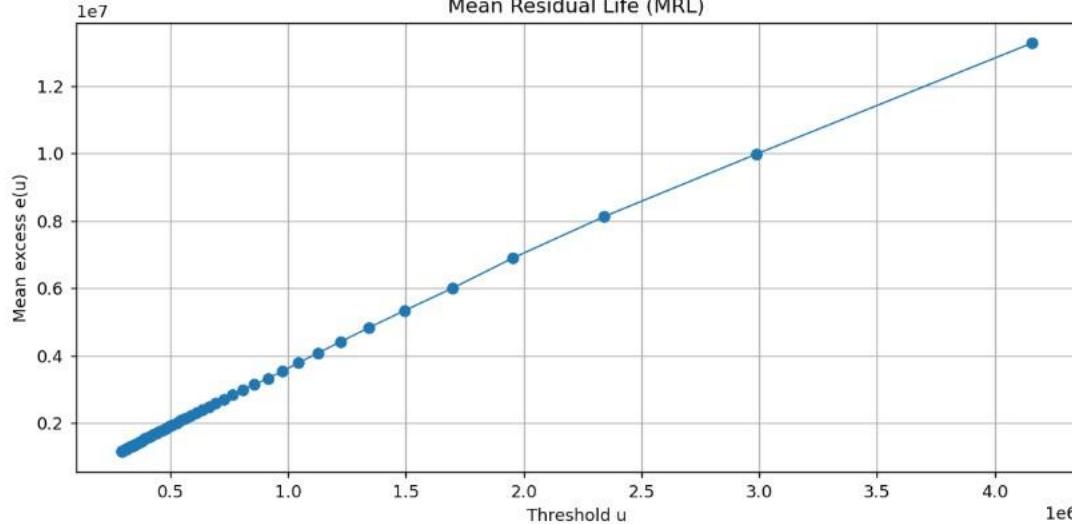


Figure 5. Mean residual life plot

Under the specification, the POT fit is heavy-tailed in which $q = 0.98$, $\hat{\xi} = 0.8254$ and $\hat{\sigma} = 1.659 \times 10^6$, and the rate of declustered exceedance is $\hat{\lambda} = 111.24$ per year. Here return levels rise rapidly with horizon (\cdot) . Using $q = 0.95$ as benchmark or comparison, $\hat{\xi} = 0.7668$ and $\hat{\sigma} = 1.576 \times 10^6$, and the rate of declustered exceedance is $\hat{\lambda} = 133.99$ per year yielded $RL_{100} = 3000000000$ showing stability and validating that there is a fat-tail in severity which means very costly but rare injuries dominate expected extreme loss. The $\hat{\sigma}$, which is the GPD scale parameter at the chosen threshold tend to be very large. Because GPD is a right-skewed distribution, a very high scale parameter shows a very spread out and fat-tailed distribution, confirming the fact that rare losses tend to have extreme financial consequences.

Method sensitivity exists when POT is compared with Block Maxima. The figure below shows that RL_{100} for POT ($q = 0.95, r = 24h$) is 3000000000 and 6700000000 for Block Maxima with annual blocks (365 D) which have a gap of around 78%. This difference is expected because Block Maxima depends on very few blocks, which inflates extrapolation uncertainty. robustness table confirms that 365D blocks sometimes yield pathological RL_{100} with small block counts, whereas semi-annual, quarterly, and monthly blocks are far more stable, giving $RL_{100} \approx 200000000$ up to $RL_{100} \approx 750000000$ across sample sizes. Conversely, once u is in the stability band, POT remains coherent across thresholds and declustering windows: ξ stays in the 0.73–0.84 range and RL_{100} sits in a tight 2700000000 up to 4750000000 as the threshold and declustering run length changes. Changes in the declustering run length influences the estimated exceedance rate, λ .

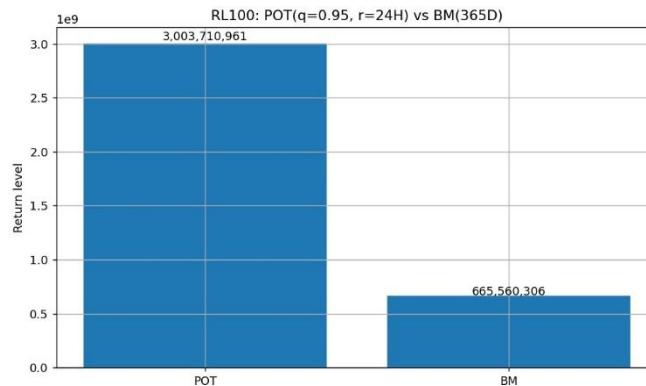


Figure 6. Return level for 100 years (peaks over threshold and block maxima)

Therefore report POT as the primary specification—anchored at $u \approx 1.0 \times 10^6$, $r \approx 24H$ —with RLs at conventional horizons and a succinct sensitivity table over q and r , and we retain BM as a cross-check using 180D and 90D blocks rather than 365D when the historical span yields few annual maxima. The practical implication for sport-injury finance is clear: the tail is heavy ($\xi \approx 0.8$), return levels escalate quickly with a multi-billion RL₁₀₀, and pricing strategies, capital allocation, and reinsurance should accommodate to extreme financial losses. Block maxima with a window of 90D-180D provides a safe benchmark & POT becomes the main estimate.

Compared to other distributions, GPD is suitable for peaks-over-threshold because it is the theoretical cap for exceedances with a high threshold – using the Pickands–Balkema–de Haan theorem any parent distribution with a high threshold approximates to GPD. It is data-efficient because it utilizes all exceedances above the threshold compared to block maxima. Furthermore, parameters are interpretable and portable too. Other alternatives include block maxima and generalized extreme value distribution and they can require lots of data and sensitive to choice of block level, in which few blocks can lead to unstable very long return levels.

4. Conclusion

This study shows that Extreme Value Theory (EVT) effectively models the heavy-tailed nature of extreme losses. The positive shape parameter showed signs of fat tails, while diagnostic tests validated the model's strong fit. Key findings include sharply increasing return levels, high conditional exceedance probabilities, and tail value-at-risk (TVaR) ruled by a small number of extreme events. These results highlight the importance of accounting for rare but severe losses in capital planning, solvency requirements, and pricing. The limitations of this study include using simulated data which may not fully reflect the real-world and it may be hard to generalize the results of this study due to limited amount of concrete data so there is not enough real evidence to create broader statements. The novelty of this paper is the extension of application to sports injury other than finance or natural disasters. Overall, EVT provides a rigorous framework for quantifying and managing extreme risk, which is robust against low-frequency, high-impact events. Future recommendations include comparing the block maxima method and peaks-over-threshold method in the context of sports injury, using Bayesian methods and comparing these methods to EVT, and using machine learning methods to model extreme financial losses in the context of sport injuries.

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